Hadronic fluctuations at non-zero density ...

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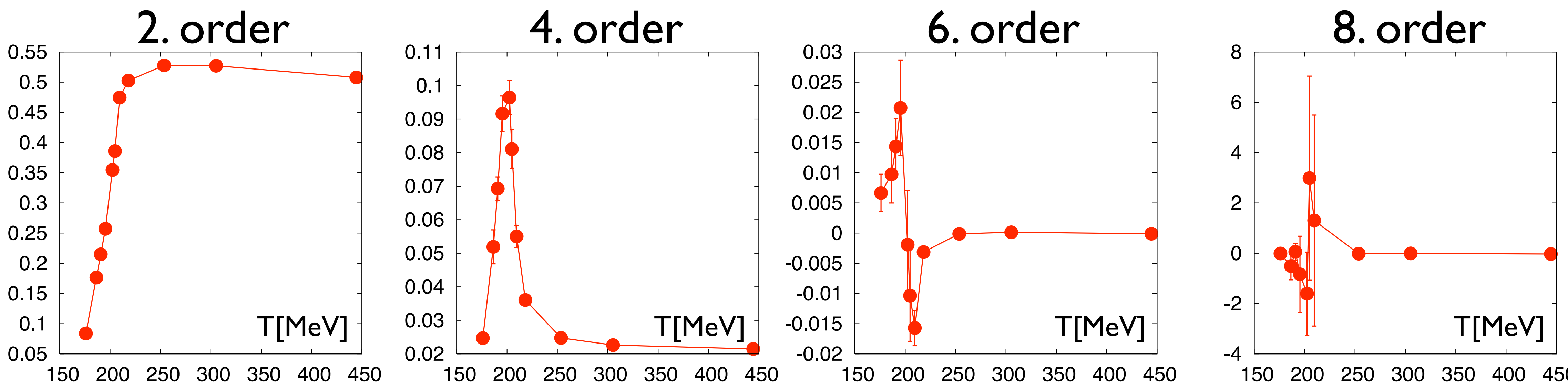
... an approximation by Taylor expansion



<u>Plan:</u>

1. Introduction to the Taylor expansion method
2. Baryon number, strangeness and electric charge fluctuations
3. Correlations among charges

Lattice QCD at non-zero temperature (and density)

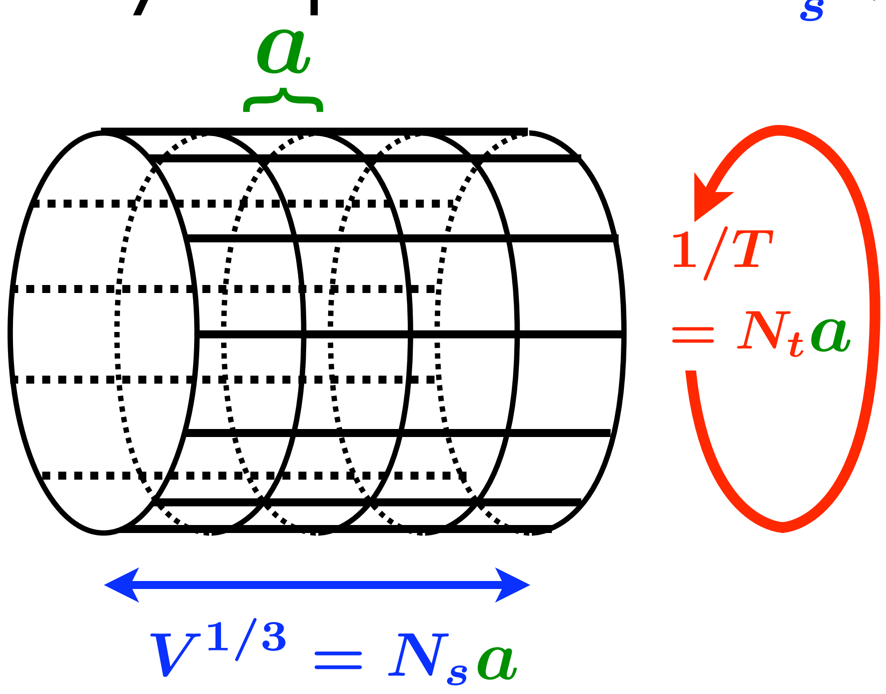
- QCD partition function:

$$Z(V,T,\mu) = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \exp\{-S_E\}$$

$$S_E = \int_0^{1/T} dx_0 \int_V d^3x \mathcal{L}_E(A,\psi,\bar{\psi},\mu)$$

$\longrightarrow$ Monte Carlo integration:
$\approx 10^6$ lattice points,
$\approx 10^8$ degrees of freedom

- Geometry of space-time: $N_s^3 \times N_t$ (4d - torus)



account for:
- finite volume effects $N_s/N_t \gtrsim 4$
- dimension 4 operatoren

$\longrightarrow$ rather large lattice spacings

$\longrightarrow$ use improved action (p4fat3)

Lattice QCD at non-zero temperature (and density)

- direct MC-simulation for $\mu > 0$ not possible

$$
\begin{aligned}
Z(V,T,\mu) &= \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \exp\{S_F(A,\psi,\bar{\psi}) - \beta S_G(A)\} \\
&= \int \mathcal{D}A\ \det[M](A,\mu) \exp\{-\beta S_G(A)\}
\end{aligned}
$$

complex for $\mu > 0$

Interpretation as probability distribution is necessary for MC-integration

$\longrightarrow$ perform a Taylor expansion around $\mu = 0$

The Taylor expansion method

- start from Taylor expansion of the pressure,

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_u,\mu_d,\mu_s) = \sum_{i,j,k} c^{u,d,s}_{i,j,k} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

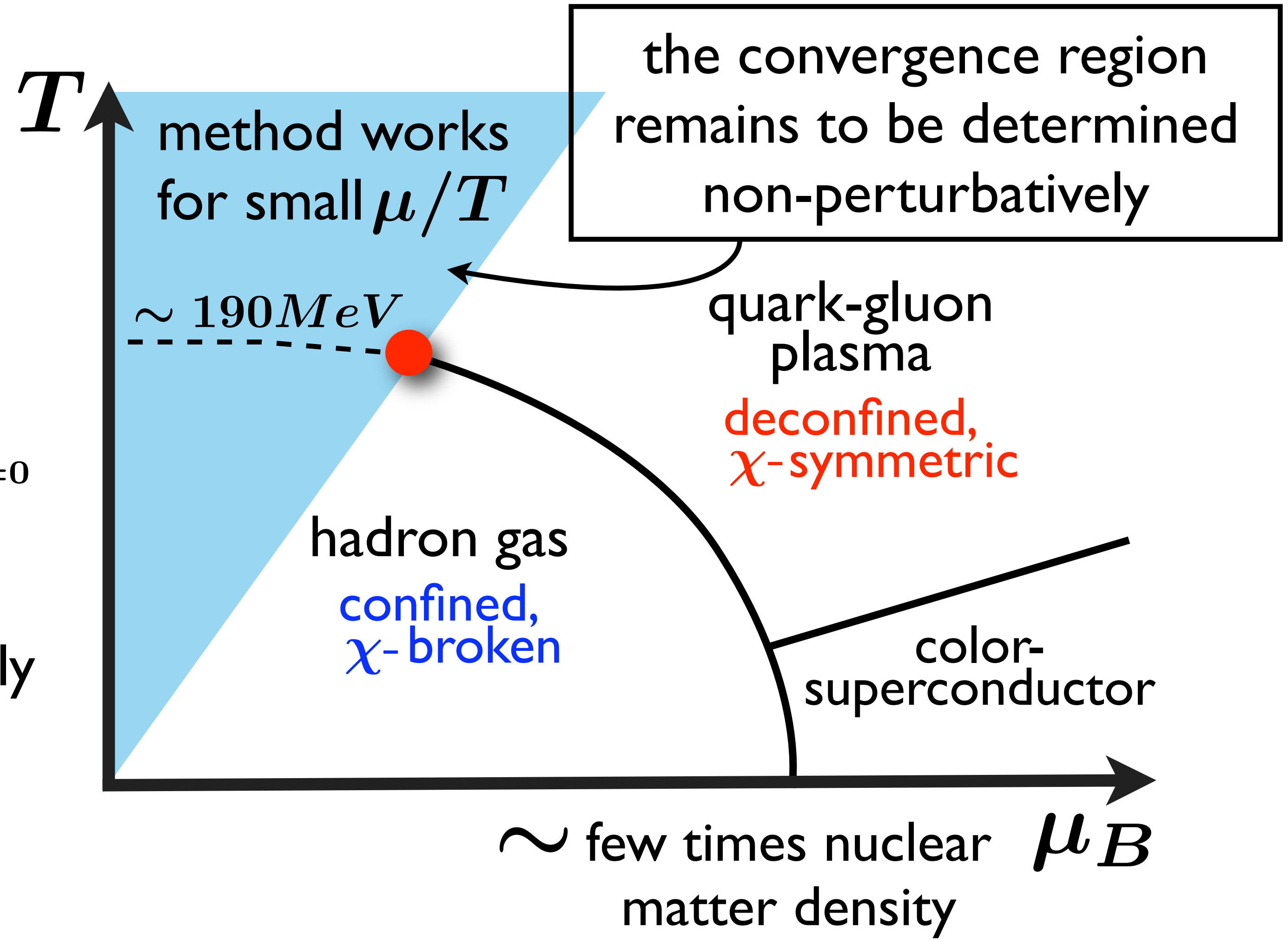
- calculate expansion coefficients for fixed temperature

- no sign problem:
  all simulations are done at $\mu = 0$

$$c^{u,d,s}_{i,j,k} \equiv \frac{1}{i!j!k!} \frac{1}{VT^3} \cdot \left. \frac{\partial^i \partial^j \partial^k \ln Z}{\partial(\frac{\mu_u}{T})^i \partial(\frac{\mu_d}{T})^j \partial(\frac{\mu_s}{T})^k} \right|_{\mu_{u,d,s}=0}$$

- method is straight forward:
  all terms can be generated automatically

Allton *et al.*, PRD66:074507,2002;
Allton *et al.*, PRD68:014507,2003;
Allton *et al.*, PRD71:054508,2005.



The Taylor expansion method

- use unbiased, noisy estimators to calculate $c_{i,j,k}^{u,d,s}$
  $\longrightarrow$ see C. Miao, CS, PoS (Lattice 2007) 175.

- line of constant physics: $m_q = m_s/10$
  (physical strange quark mass)

- measure currently up to
$$\mathcal{O}(\mu^8) \longleftrightarrow (N_t = 4)$$
$$\mathcal{O}(\mu^4) \longleftrightarrow (N_t = 6)$$

- expansion coefficients $c_{i,j,k}^{u,d,s}$ are related to B,S,Q-fluctuations

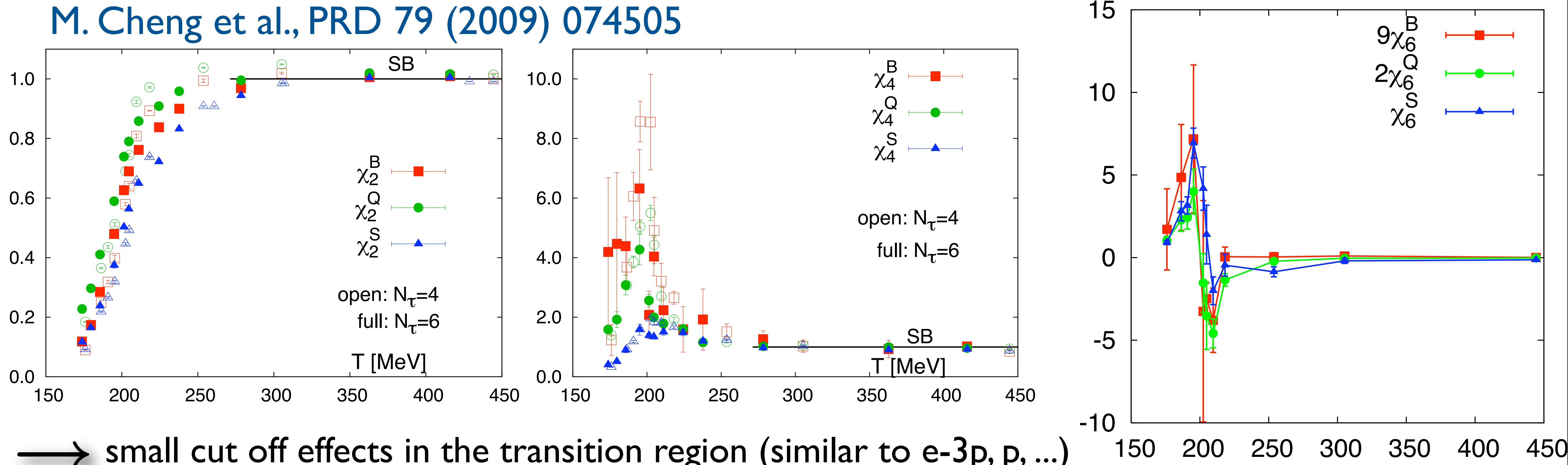
$$\begin{aligned} n_B &= \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \frac{1}{3}(n_u + n_d + n_s) \\ n_S &= \frac{\partial(p/T^4)}{\partial(\mu_S/T)} = -n_s \\ n_Q &= \frac{\partial(p/T^4)}{\partial(\mu_Q/T)} = \frac{1}{3}(2n_u - n_d - n_s) \end{aligned}$$

$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S \end{aligned}$$

- choice of $\mu_u \equiv \mu_d$ is equivalent to $\mu_Q \equiv 0$

B,Q,S fluctuations (at $\mu = 0$)

M. Cheng et al., PRD 79 (2009) 074505



→ small cut off effects in the transition region (similar to e-3p, p, ...)

→ general pattern can be understood by the singular behavior of the free energy

$$\chi_{2n}^B \sim \left| \frac{T - T_c}{T_c} \right|^{2-n-\alpha} , \qquad \alpha \approx -0.25$$

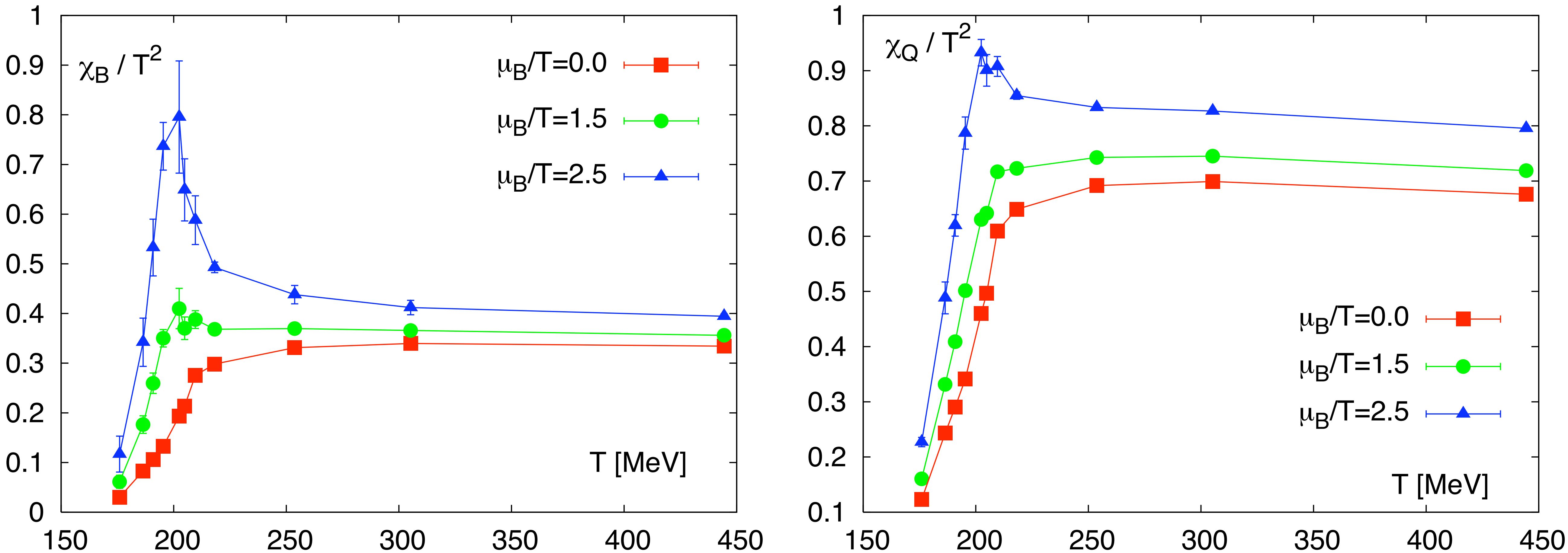
χ_2^B dominated by the regular part, χ_4^B develops a cusp.

- We define fluctuations of charge X as

$$\begin{aligned}
\chi_2^X &= \frac{1}{VT^3} \langle N_X^2 \rangle &&= 2!\ c_2^X \\
\chi_4^X &= \frac{1}{VT^3} \left(\langle N_X^4 \rangle - \langle N_X^2 \rangle^2 \right) &&= 4!\ c_4^X \\
\chi_6^X &= \frac{1}{VT^3} \left(\langle N_X^6 \rangle - 15 \langle N_X^4 \rangle \langle N_X^2 \rangle + 30 \langle N_X^2 \rangle^3 \right) &&= 6!\ c_6^X
\end{aligned}$$

B,Q,S fluctuations (at $\mu > 0$)

$$\frac{\chi_B}{T^2} = 2c_2^B + 12c_4^B\left(\frac{\mu_B}{T}\right)^2 \qquad \frac{\chi_Q}{T^2} = 2c_2^Q + 2c_{22}^{BQ}\left(\frac{\mu_B}{T}\right)^2$$

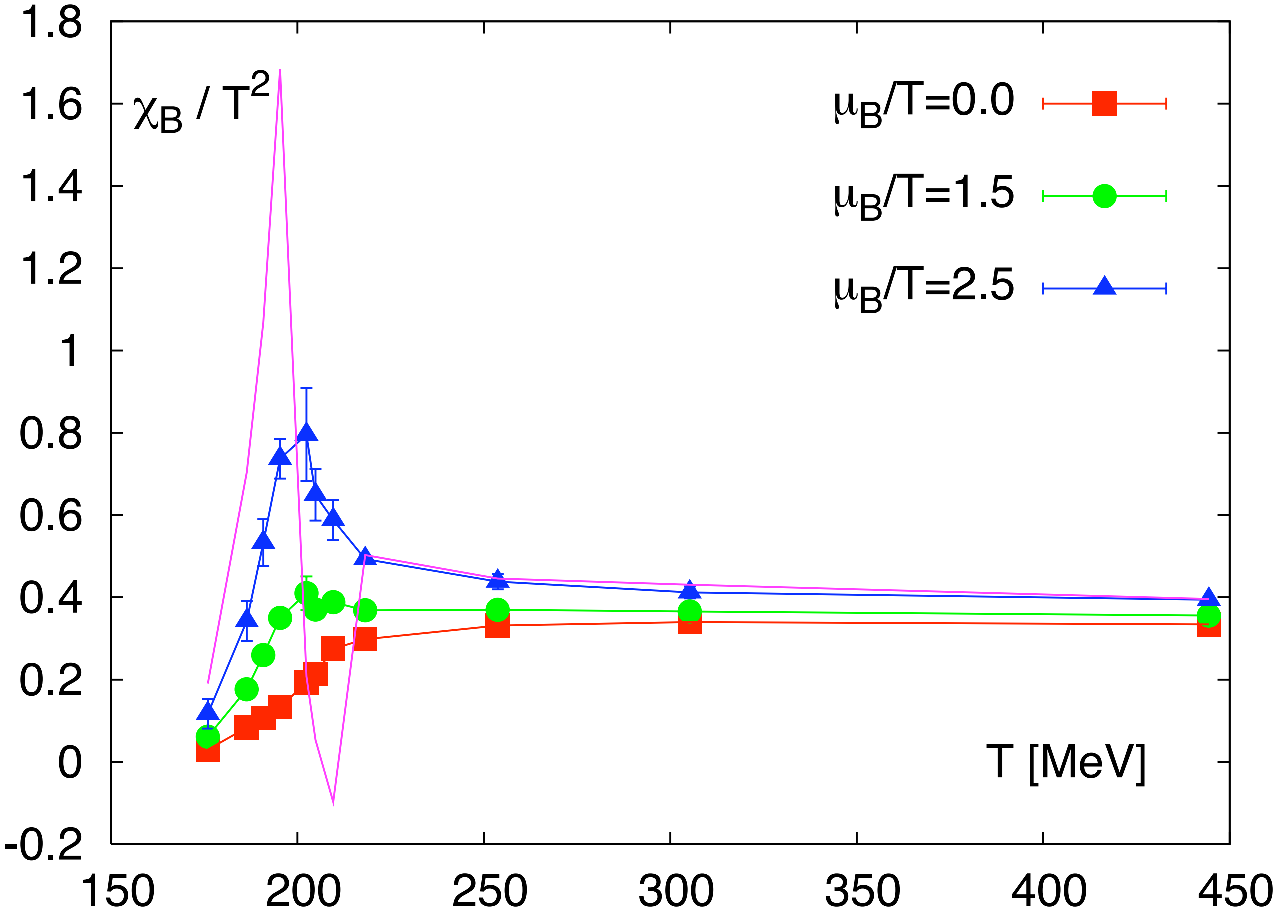


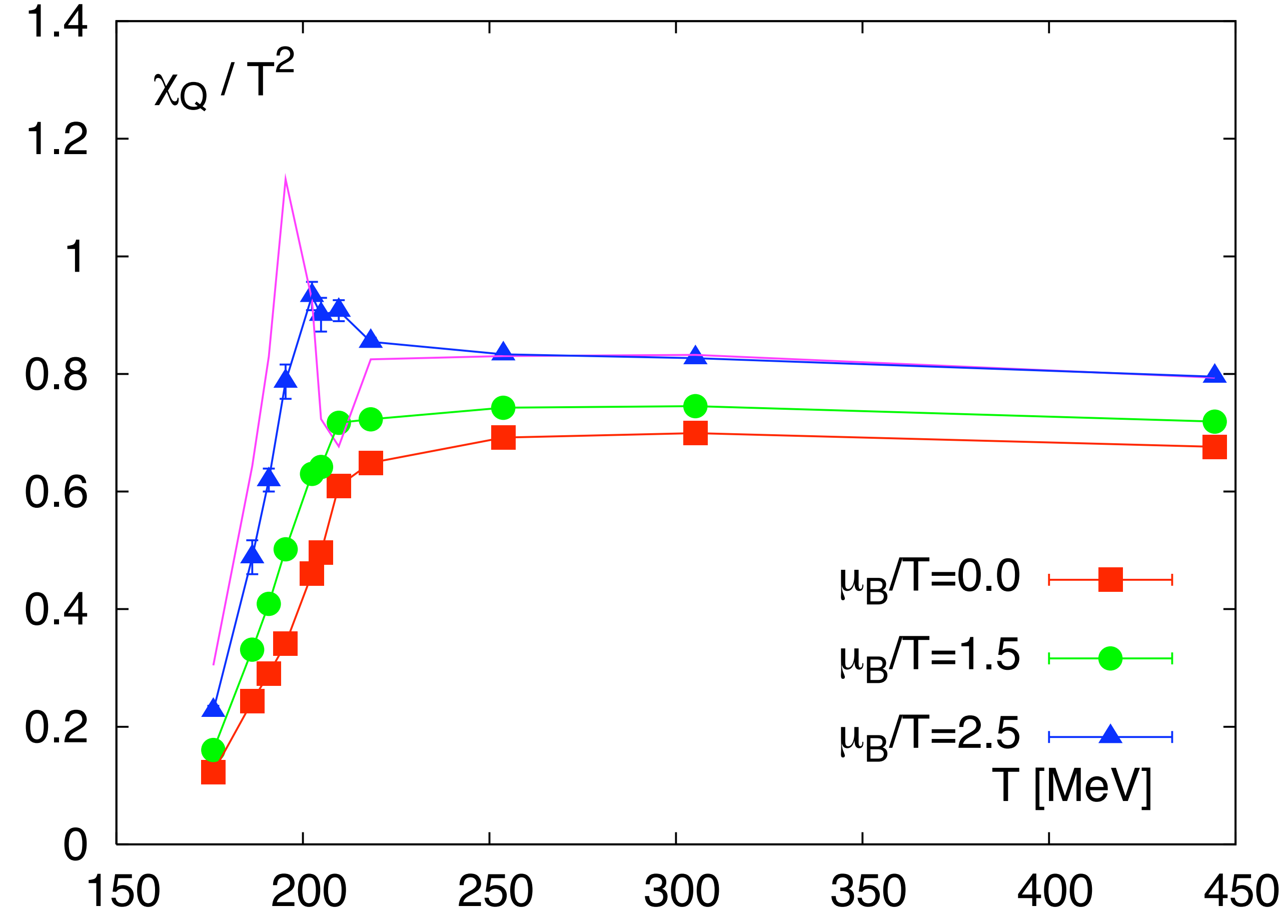
$\longrightarrow$ **evidence for a critical point ?**

Seeing „true“ singular behavior as a signal for a critical point requires large volumes and high order Taylor expansions

$$\frac{\chi_B}{T^2} = 2c_2^B + 12c_4^B \left(\frac{\mu_B}{T}\right)^2 + 30c_6^B \left(\frac{\mu_B}{T}\right)^4$$

$$\frac{\chi_Q}{T^2} = 2c_2^B + 2c_{22}^{BQ} \left(\frac{\mu_B}{T}\right)^2 + 2c_{42}^{BQ} \left(\frac{\mu_B}{T}\right)^4$$





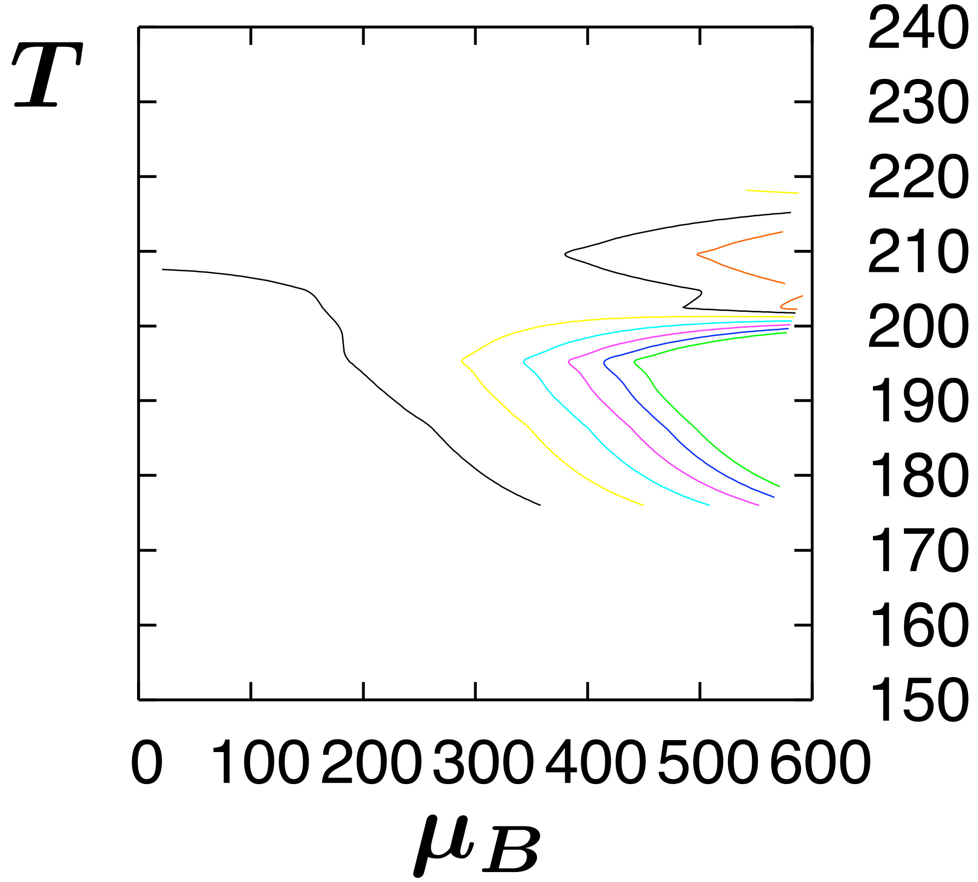
⟶ **evidence for a critical point ?**

Seeing „true“ singular behavior as a signal for a critical point requires large volumes and high order Taylor expansions

Contour plots:

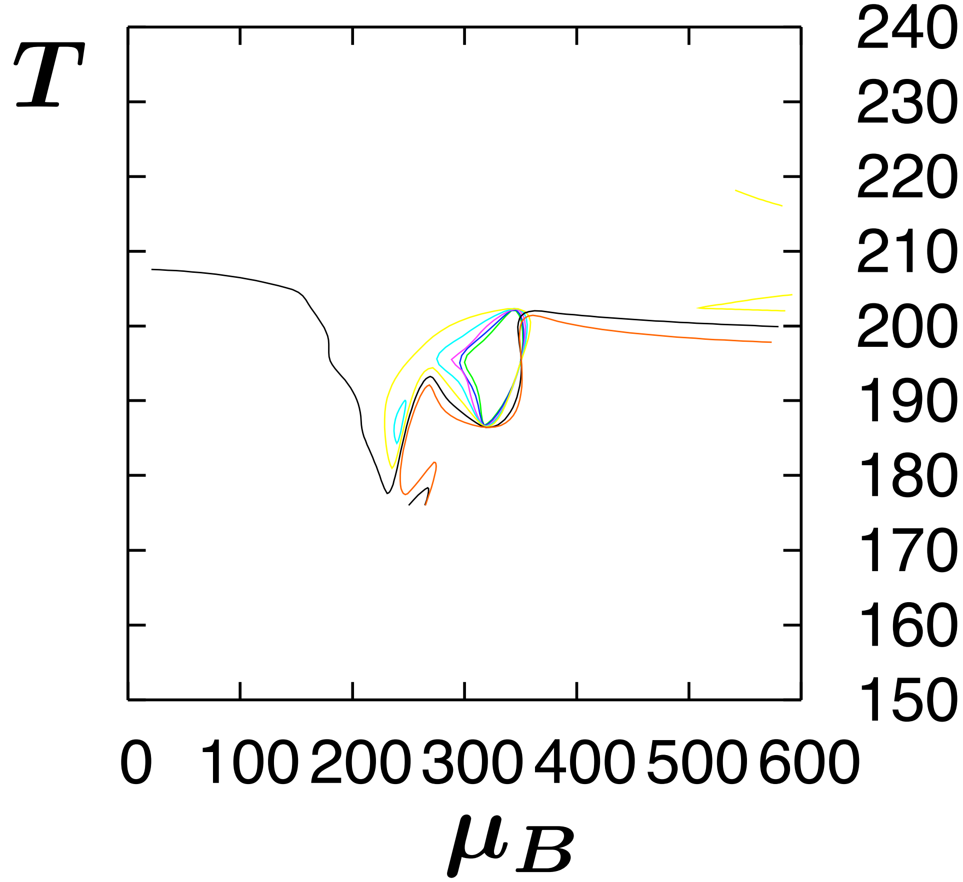
Taylor 6th order:

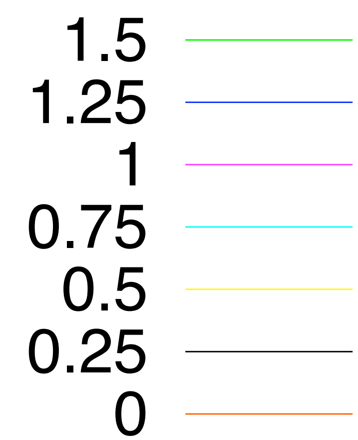
$$\frac{\chi_B}{T^2} = 2c_2^B + 12c_4^B \left(\frac{\mu_B}{T}\right)^2 + 30c_6^B \left(\frac{\mu_B}{T}\right)^4$$



Pade [2,2]:

$$\frac{\chi_B}{T^2} = \frac{2c_2c_4 + (12c_4^2 - 5c_2c_6)\left(\frac{\mu_B}{T}\right)^2}{c_4 - \frac{5}{2}c_6\left(\frac{\mu_B}{T}\right)^2}$$

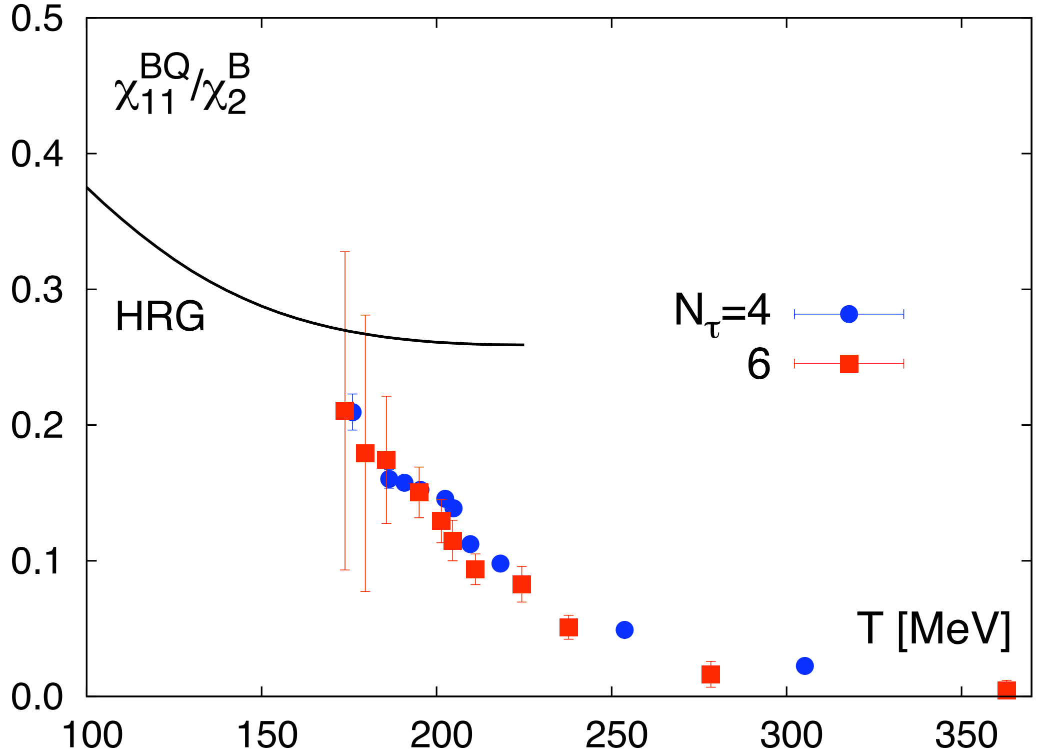


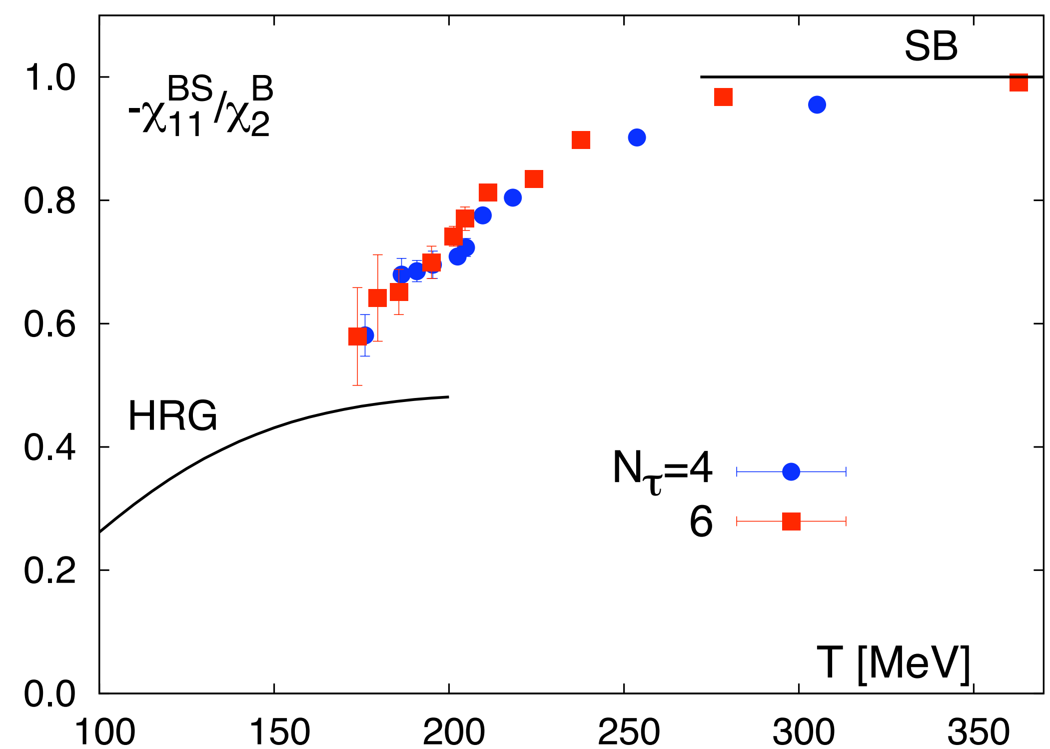


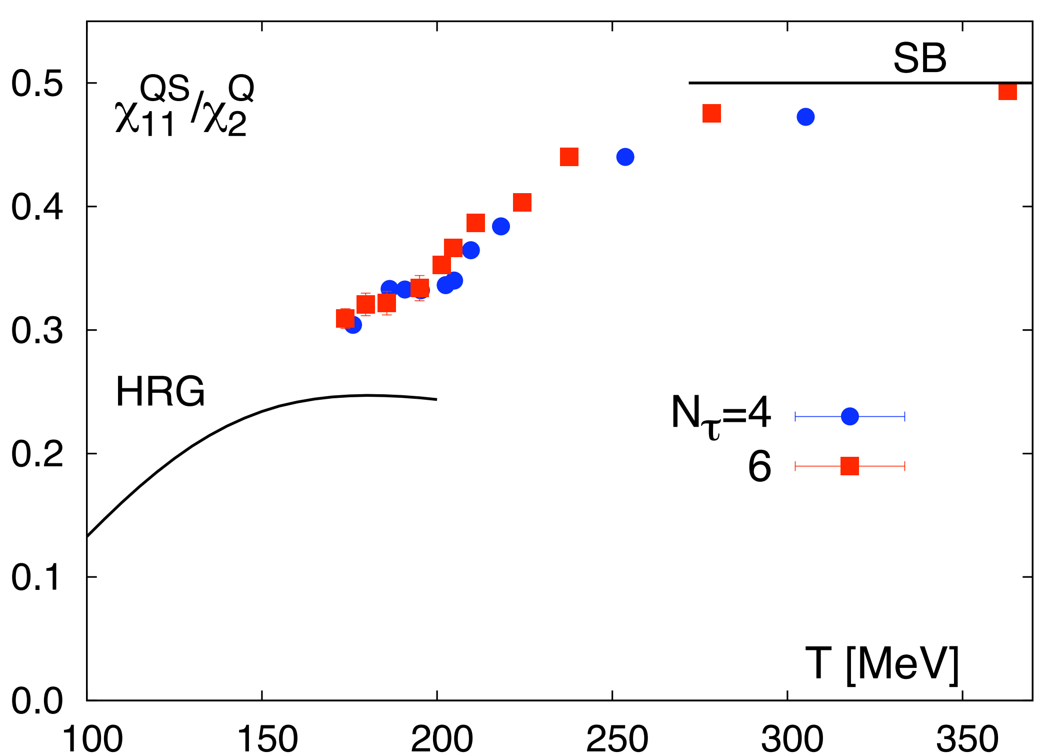
$\longrightarrow$ **does resummation of coefficients help ?**

similar results have been obtained by Gavai, Gupta, PRD 78 (2008)114503.

Hadronic Correlations (at $\mu = 0$)







$\longrightarrow$ agreement with free gas results for T>1.5 Tc

$\longrightarrow$ qualitative agreement with the resonance gas below Tc

$\longrightarrow$ small cut-off effects

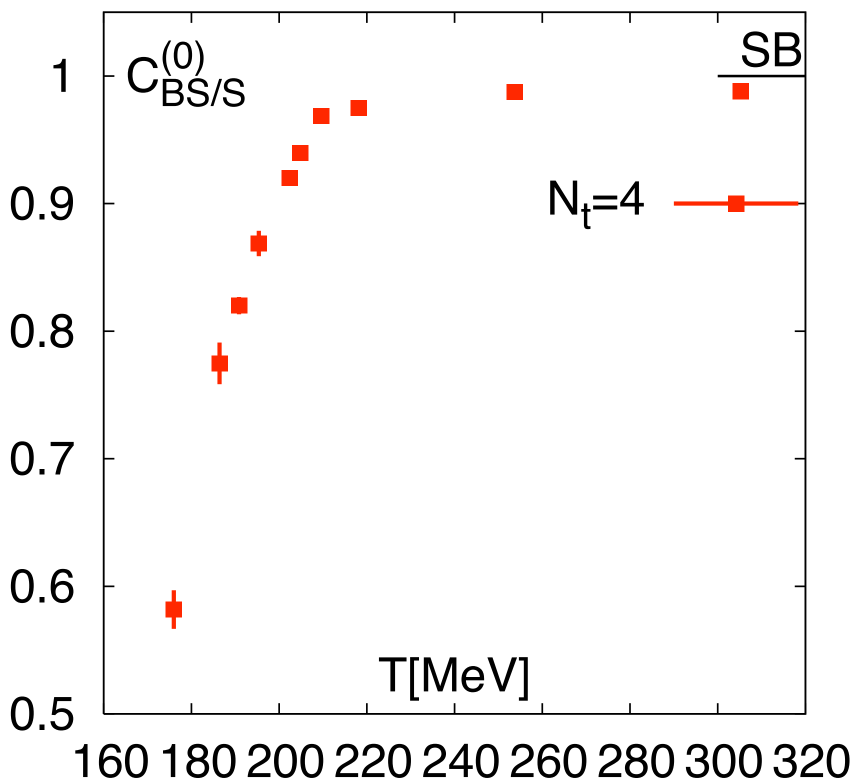
Hadronic Correlations (at $\mu > 0$)

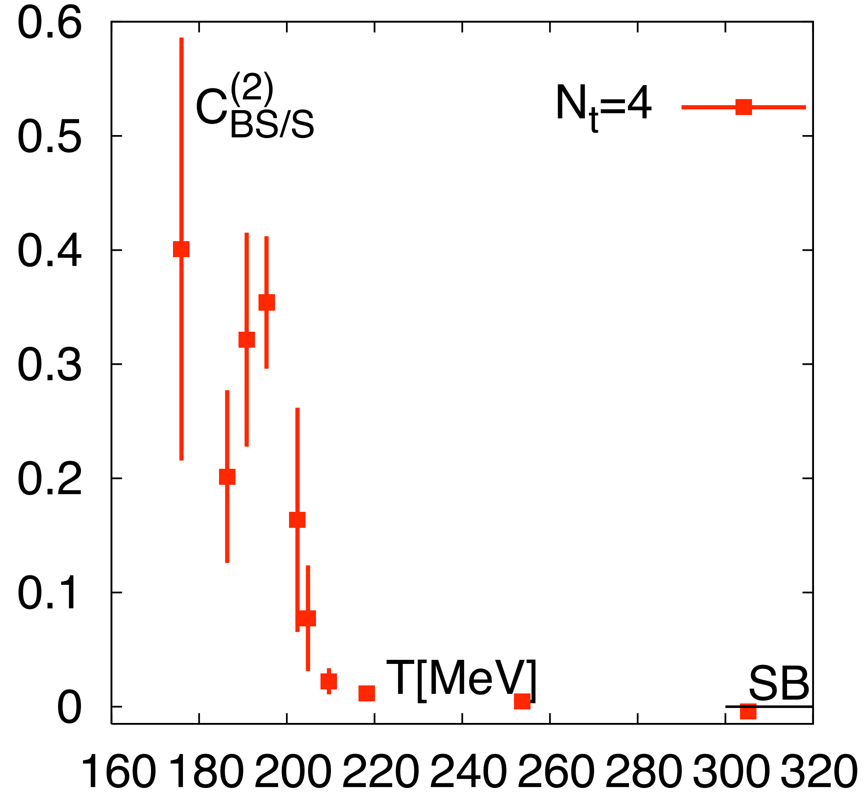
$$C_{BS/S} \equiv -3\frac{\chi_{11}^{BS}}{\chi_2^S} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2}$$

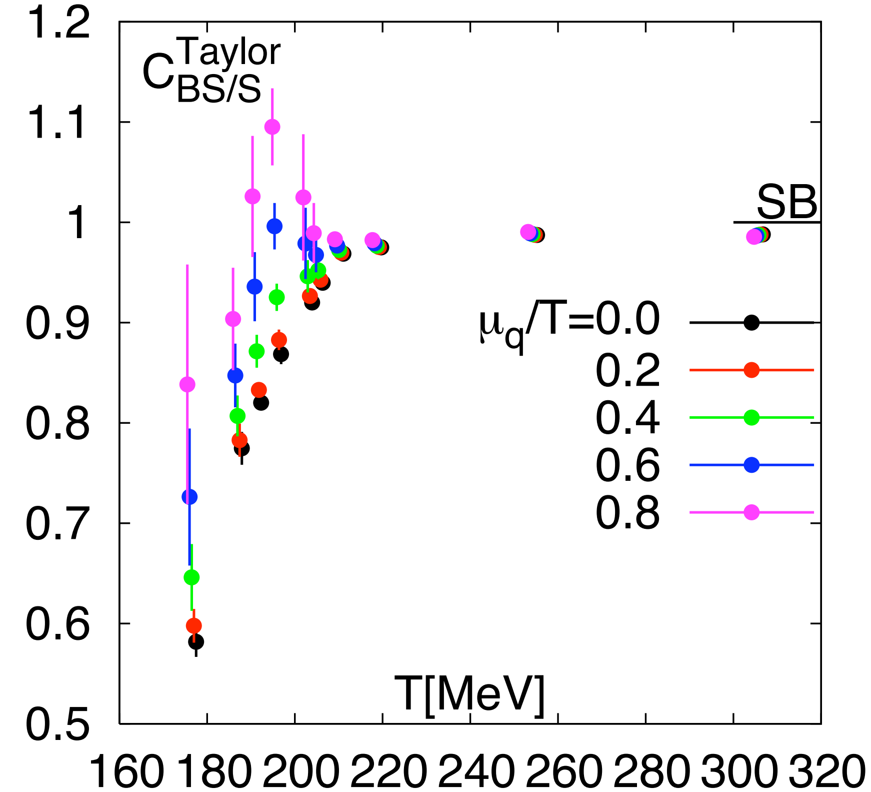
Koch, Majumder, Randrup (`05)

Taylor expansion in μ_q/T with $\frac{\partial}{\partial\mu_q} = \left[\frac{\partial}{\partial\mu_u} + \frac{\partial}{\partial\mu_d}\right]$

$$C^{(0)}_{BS/S} = 1 + \frac{c^{q,s}_{1,1}}{2c^{q,s}_{0,2}} \qquad C^{(2)}_{BS/S} = \frac{-c^{q,s}_{1,1}c^{q,s}_{2,2} + c^{q,s}_{0,2}c^{q,s}_{3,1}}{2c^{q,s\,2}_{0,2}} \qquad C^{\text{Taylor}}_{BS/S} = C^{(0)}_{BS/S} + C^{(2)}_{BS/S}\left(\frac{\mu_q}{T}\right)^2$$







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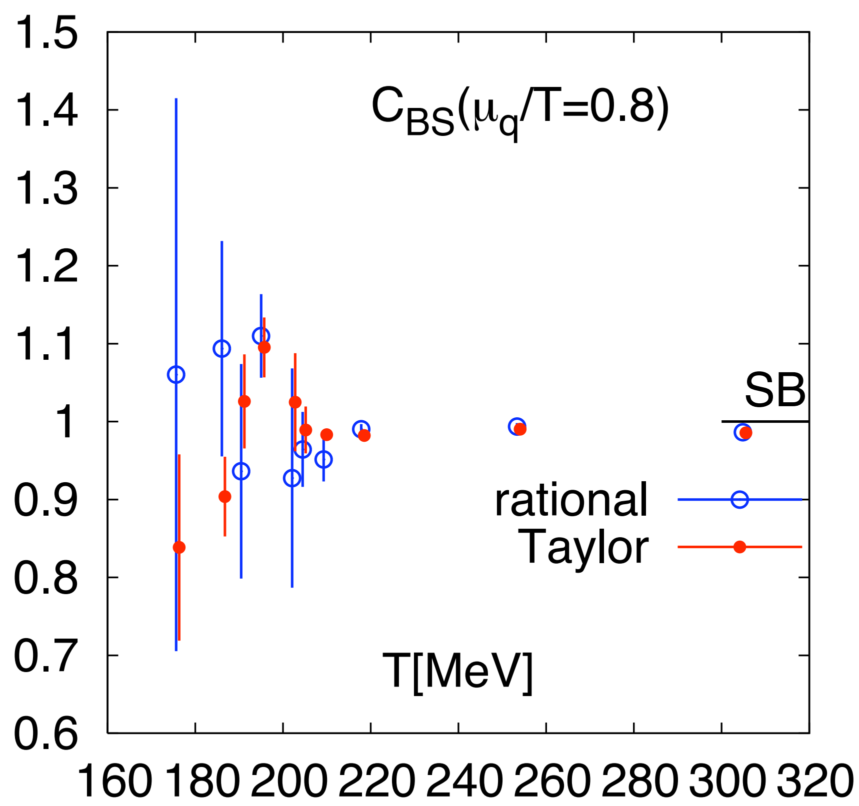
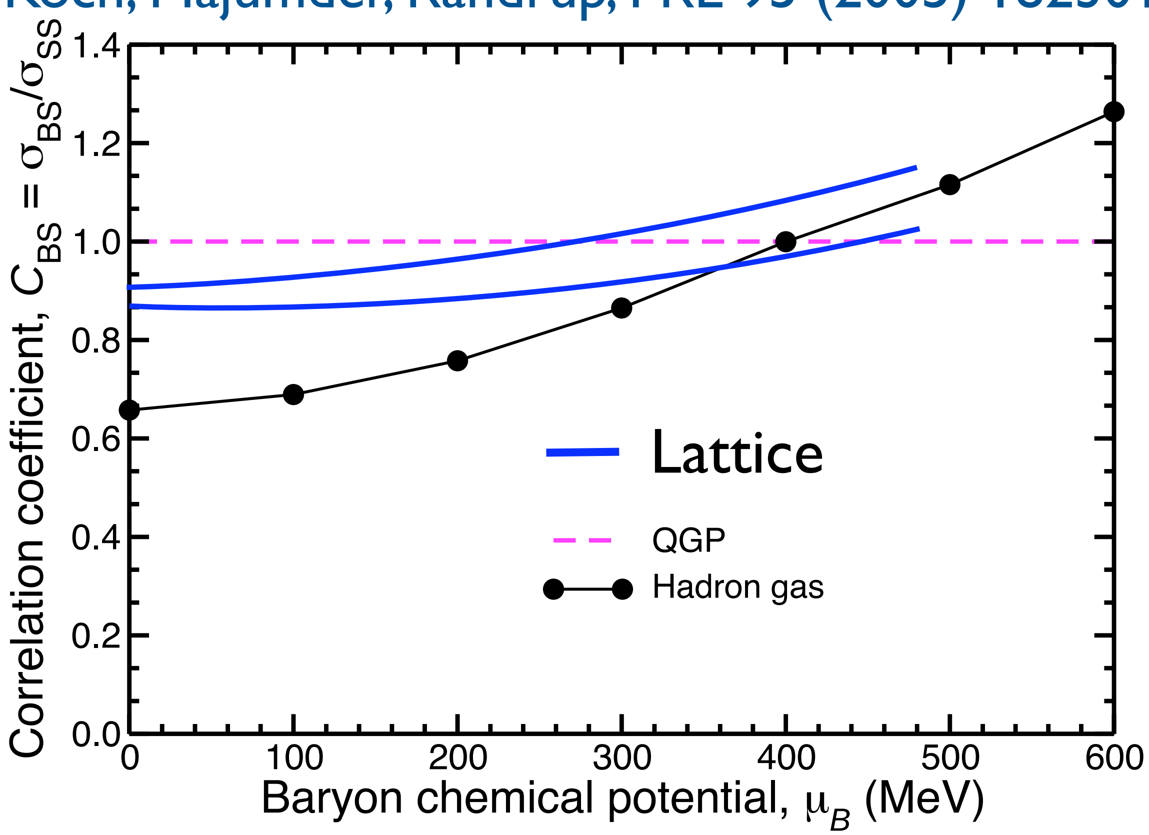
Taylor expansion in μ_q/T with $\frac{\partial}{\partial\mu_q} = \left[\frac{\partial}{\partial\mu_u} + \frac{\partial}{\partial\mu_d}\right]$

$$C_{BS}^{\text{rational}} = 1 + \frac{c_{1,1}^{q,s} + c_{3,1}^{q,s}(\mu_q/T)^2 + c_{5,1}^{q,s}(\mu_q/T)^4}{2c_{0,2}^{q,s} + 2c_{2,2}^{q,s}(\mu_q/T)^2 + 2c_{4,2}^{q,s}(\mu_q/T)^4}$$



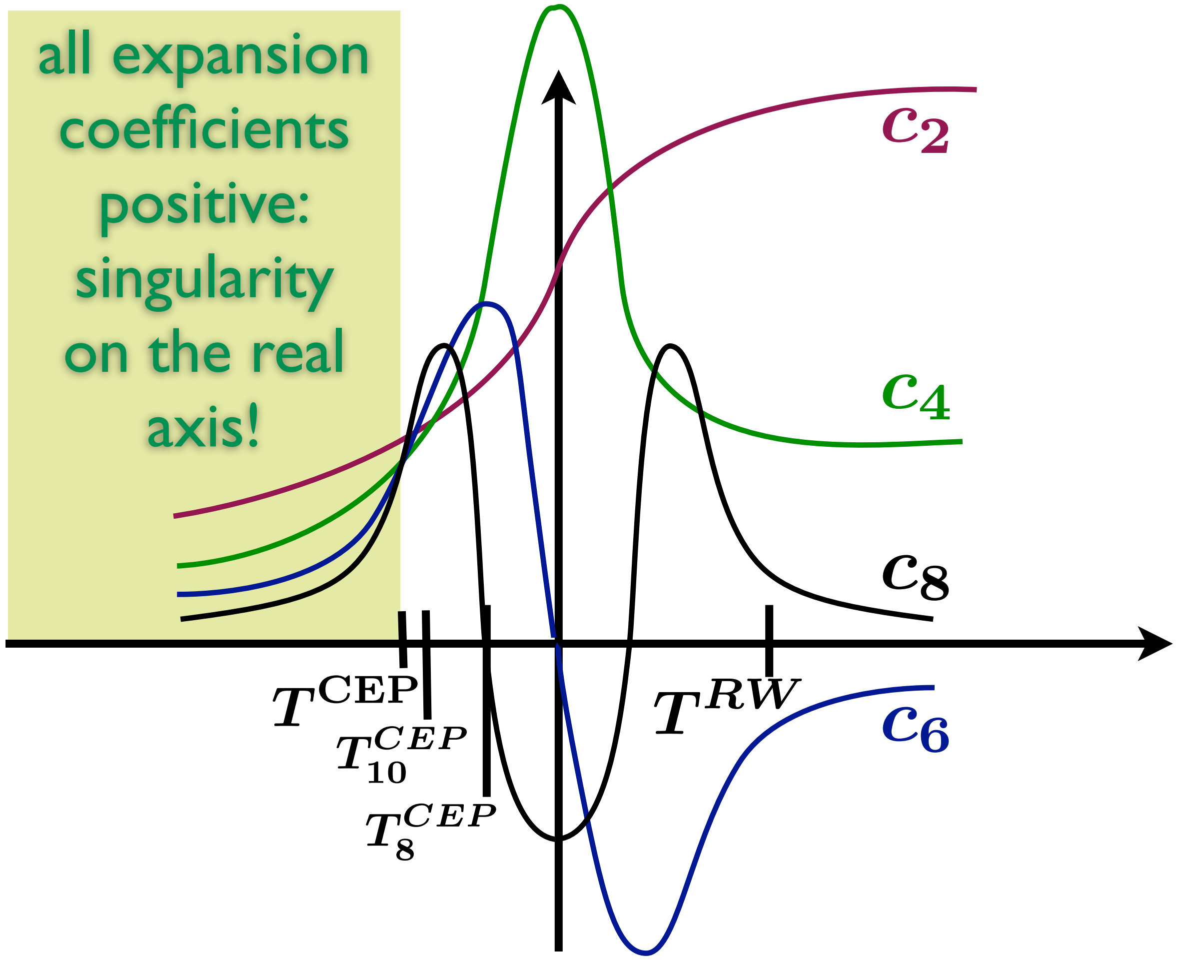
Koch, Majumder, Randrup, PRL 95 (2005) 182301

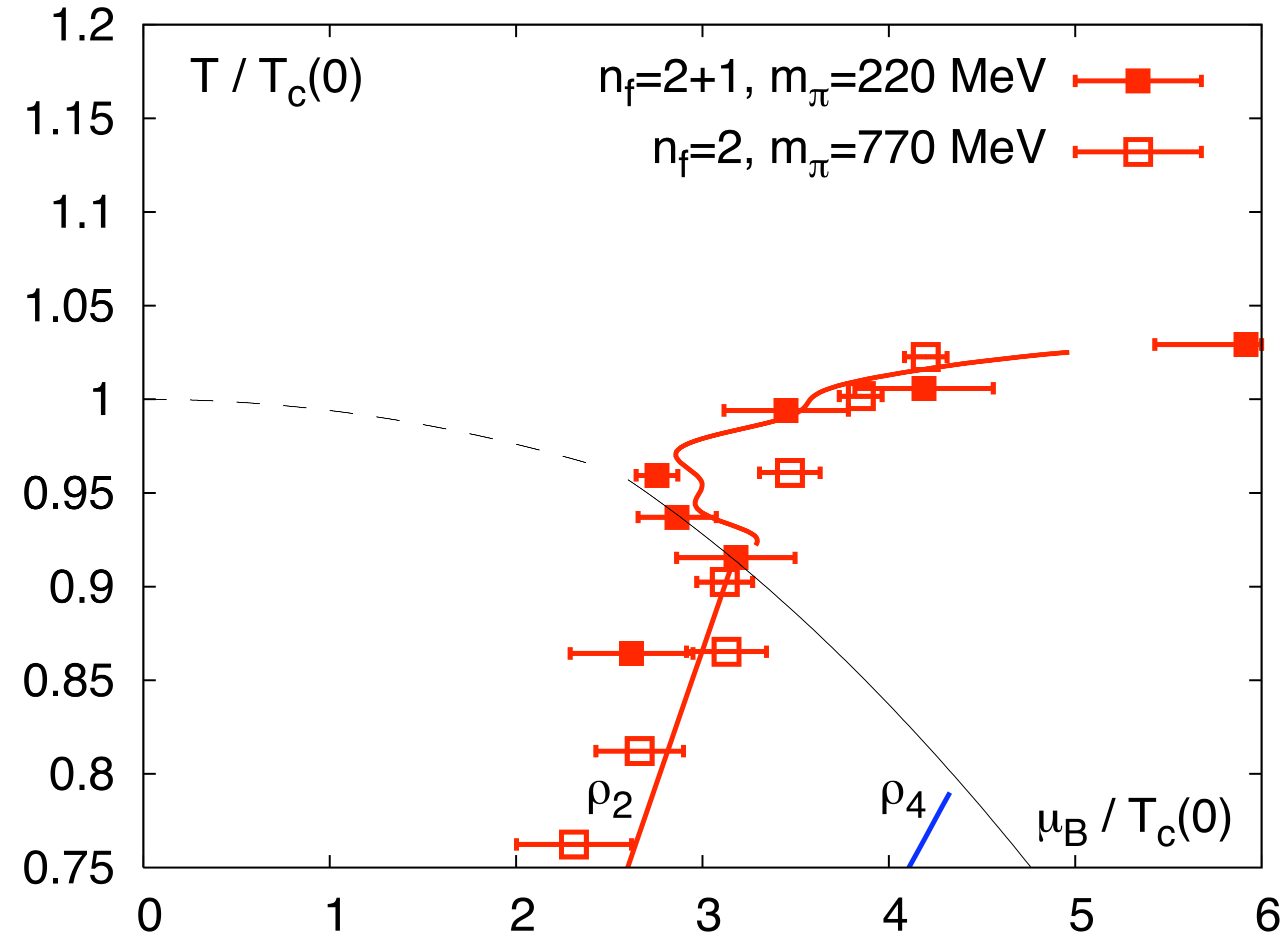


Radius of Convergence

Method to determine the CEP:

- find largest temperature were all expansion coefficients are positive → T^{CEP}
- determine the radius of convergence at that temperature → μ^{CEP}





$$\mu^{\mathrm{CEP}} = \lim_{n\to\infty} \mu_n^{\mathrm{CEP}}$$

$$\mu_n^{\mathrm{CEP}} = T^{\mathrm{CEP}} \sqrt{c_n/c_{n+2}}$$

→ first non-trivial estimate of T^{CEP} from c_8
second non-trivial estimate of T^{CEP} from c_{10}

The Resonance gas limit:

$$\frac{p}{T^4} = G(T) + F(T)\cosh\left(\frac{\mu_B}{T}\right)$$

$$\longrightarrow \rho_n = \sqrt{(n+2)(n+1)}$$

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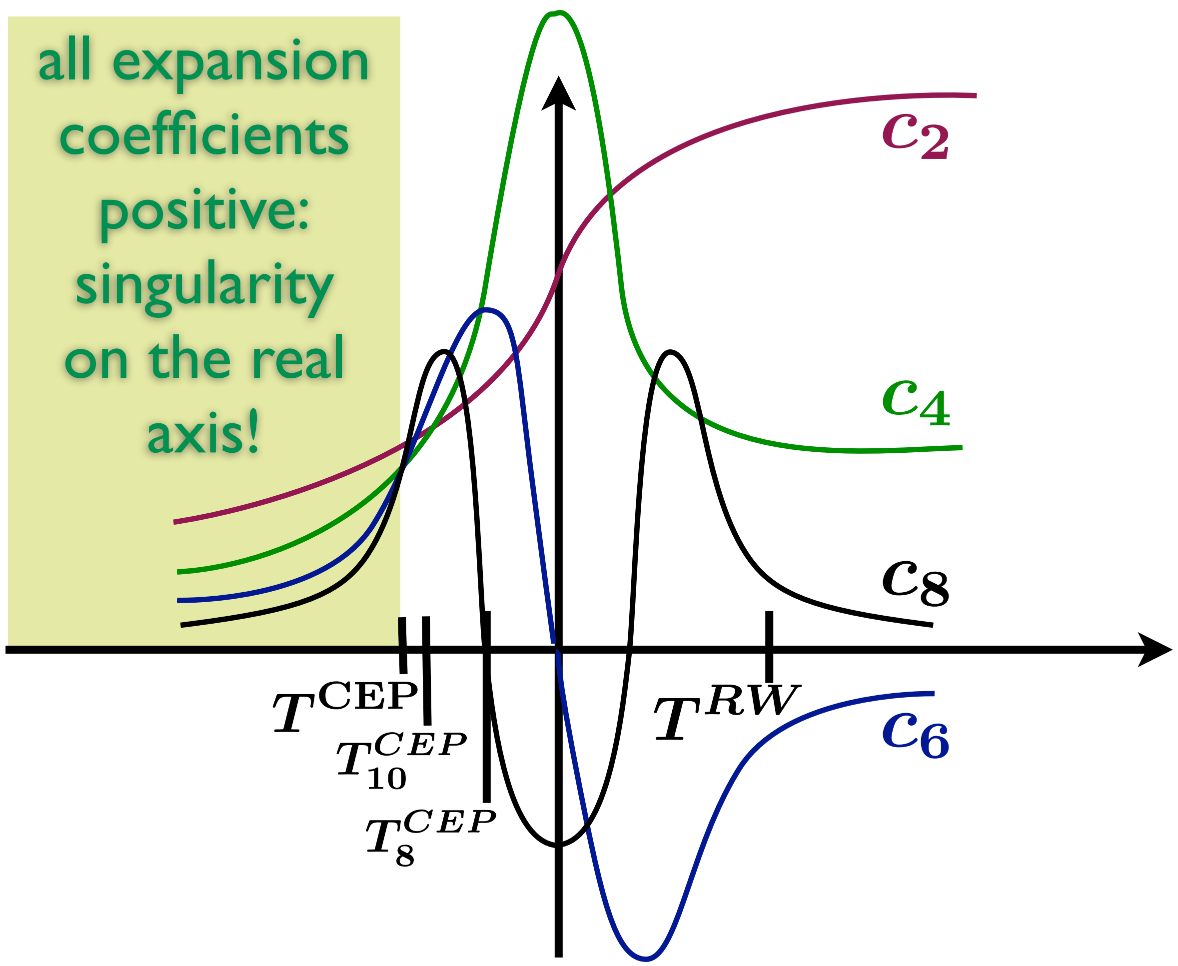

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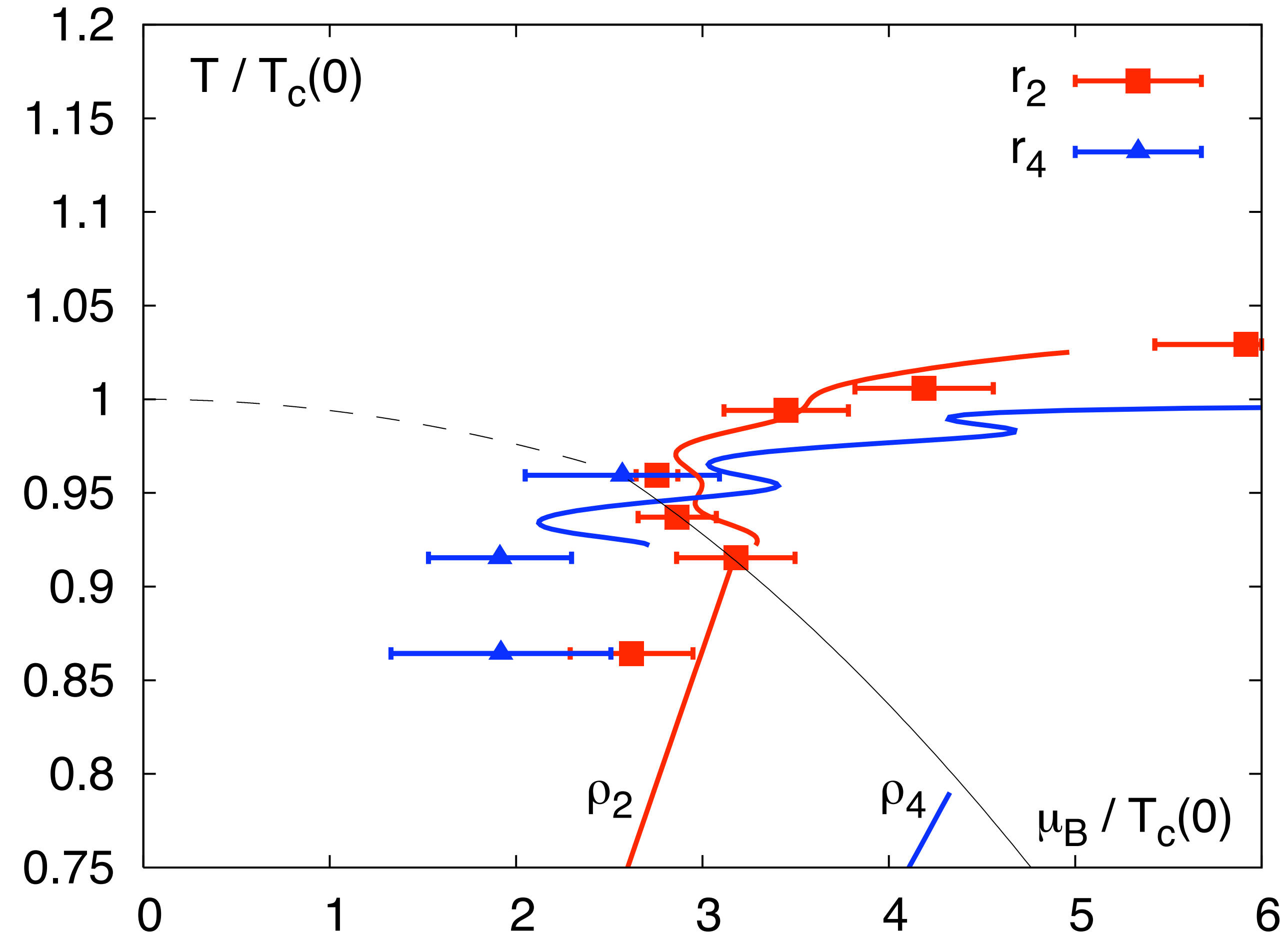

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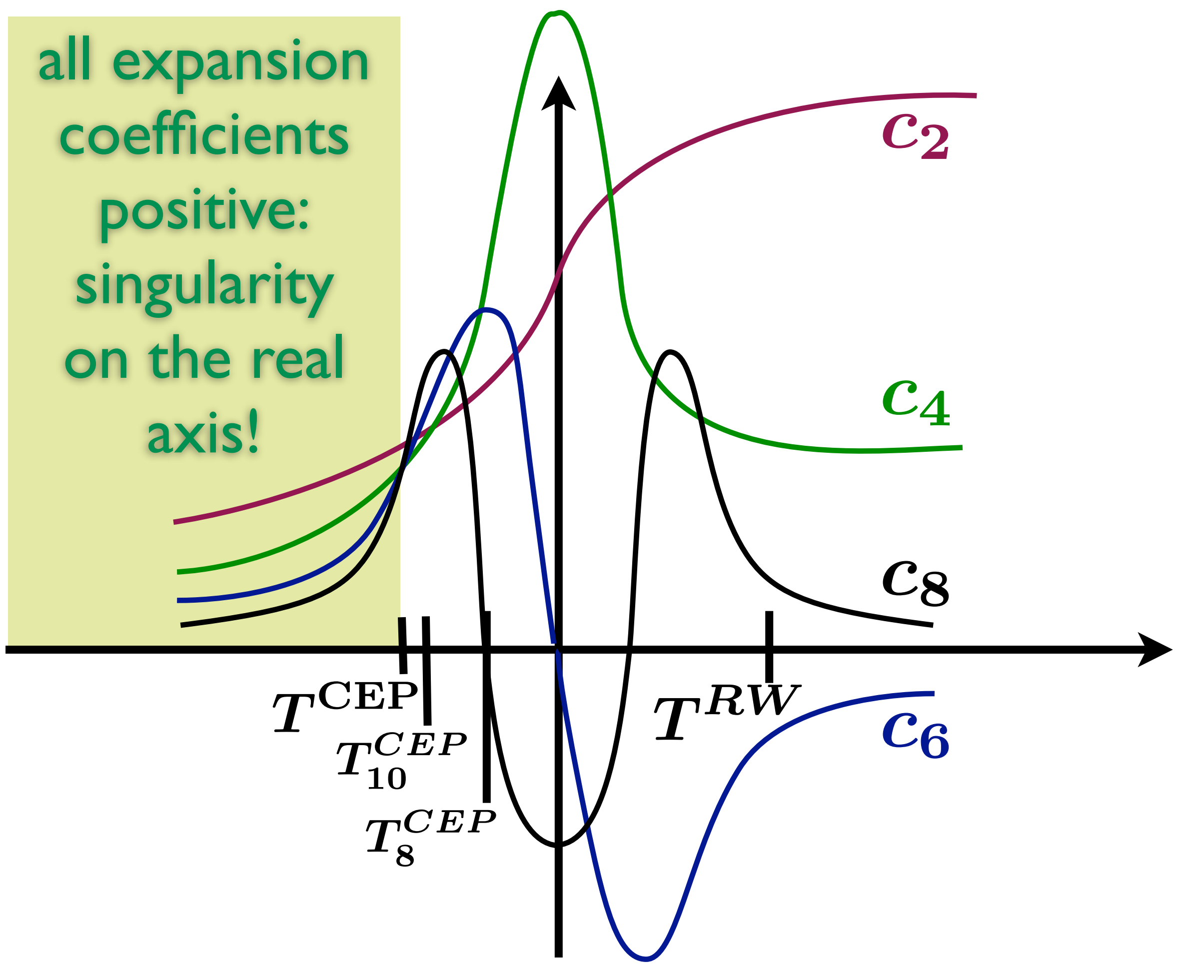

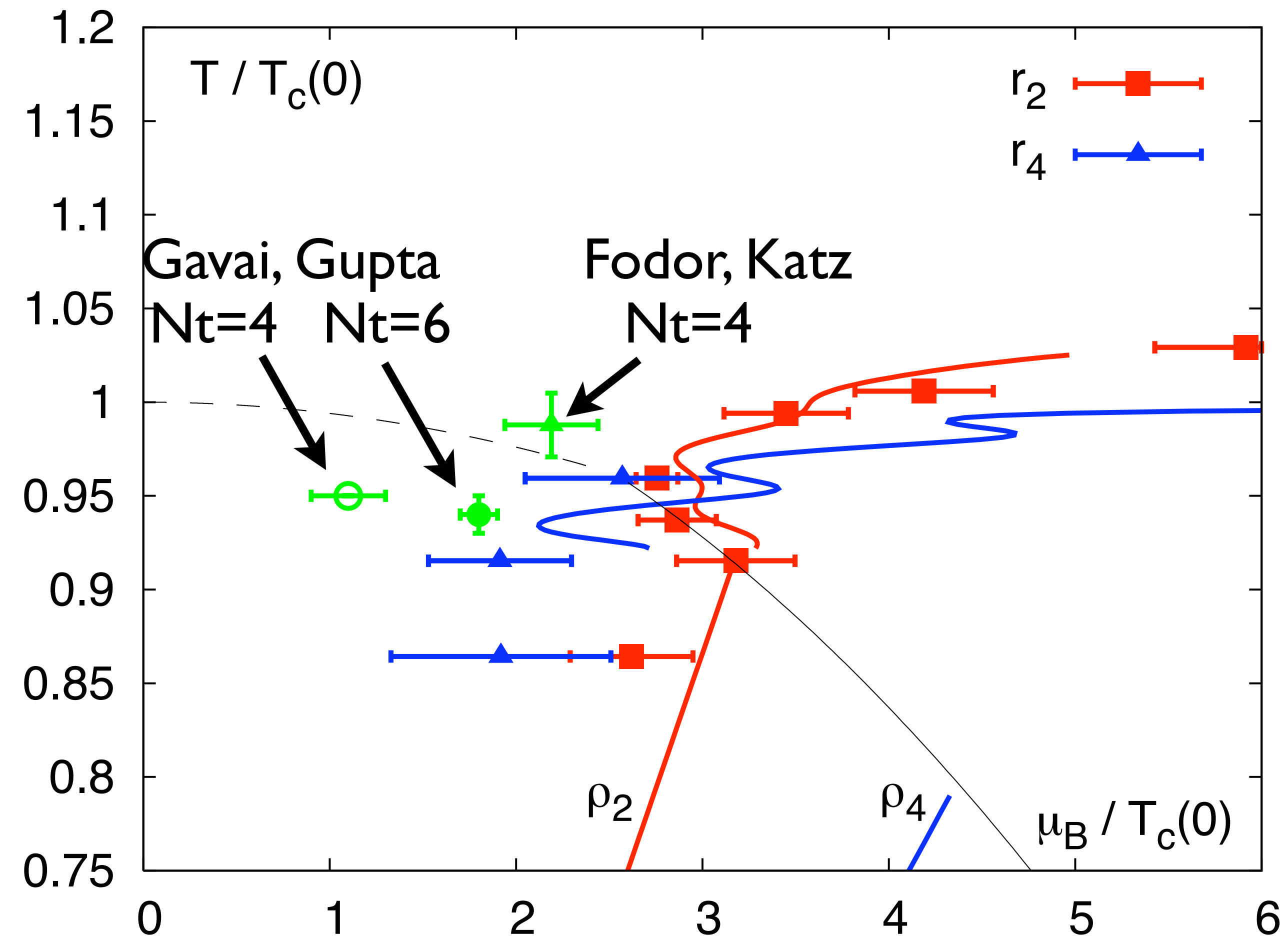

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Summary

- Taylor expansion coefficients show small cutoff effects (with improved p4fat3 action)

- partial sums of Taylor expansion have to be taken with care, a re-summation (Pade) might help

- second order expansion of $C_{BS/S}$ indicate growth of correlations below Tc (compatible with HRG)

- the radius of convergence determines the position of the CEP

- including 6th order in the approximation of the convergence radius will decrease approximations for μ_B^{CEP} and T^{CEP}

- warning: shown results mostly $N_\tau = 4$ and masses are not physical

Hadronic Correlations (at $\mu > 0$)

$$C_{BS/S} \equiv -3\frac{\chi_{11}^{BS}}{\chi_2^S} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2}$$

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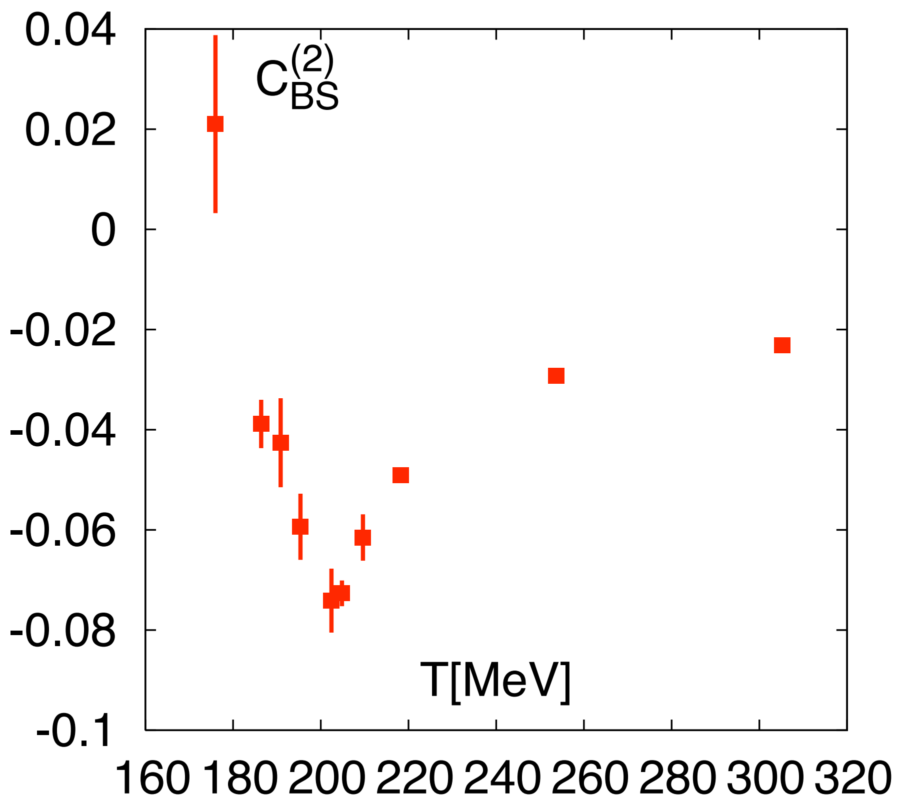
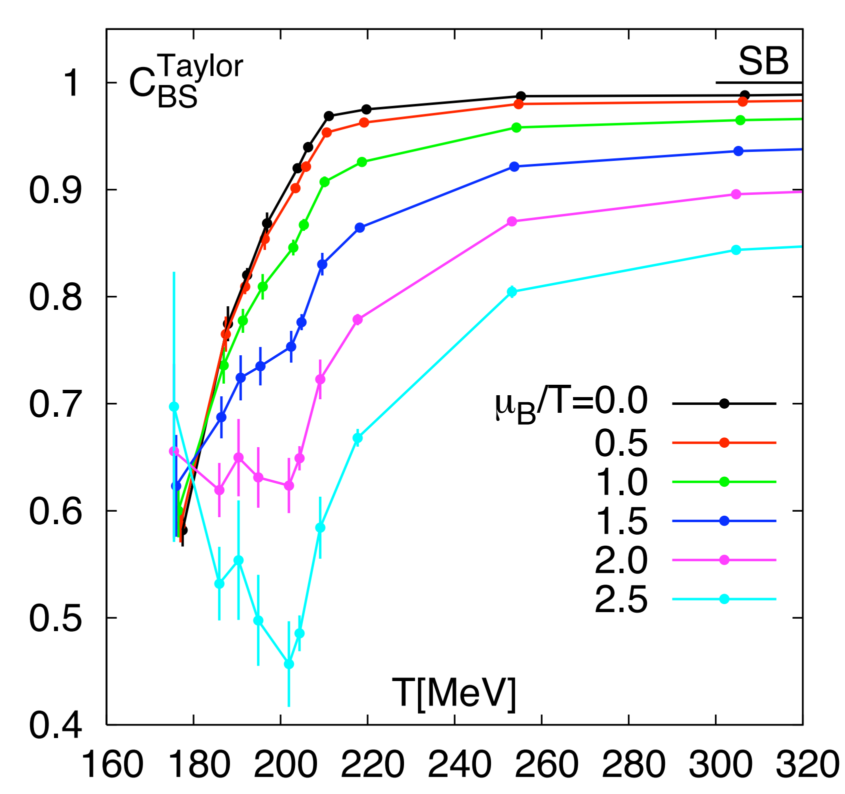
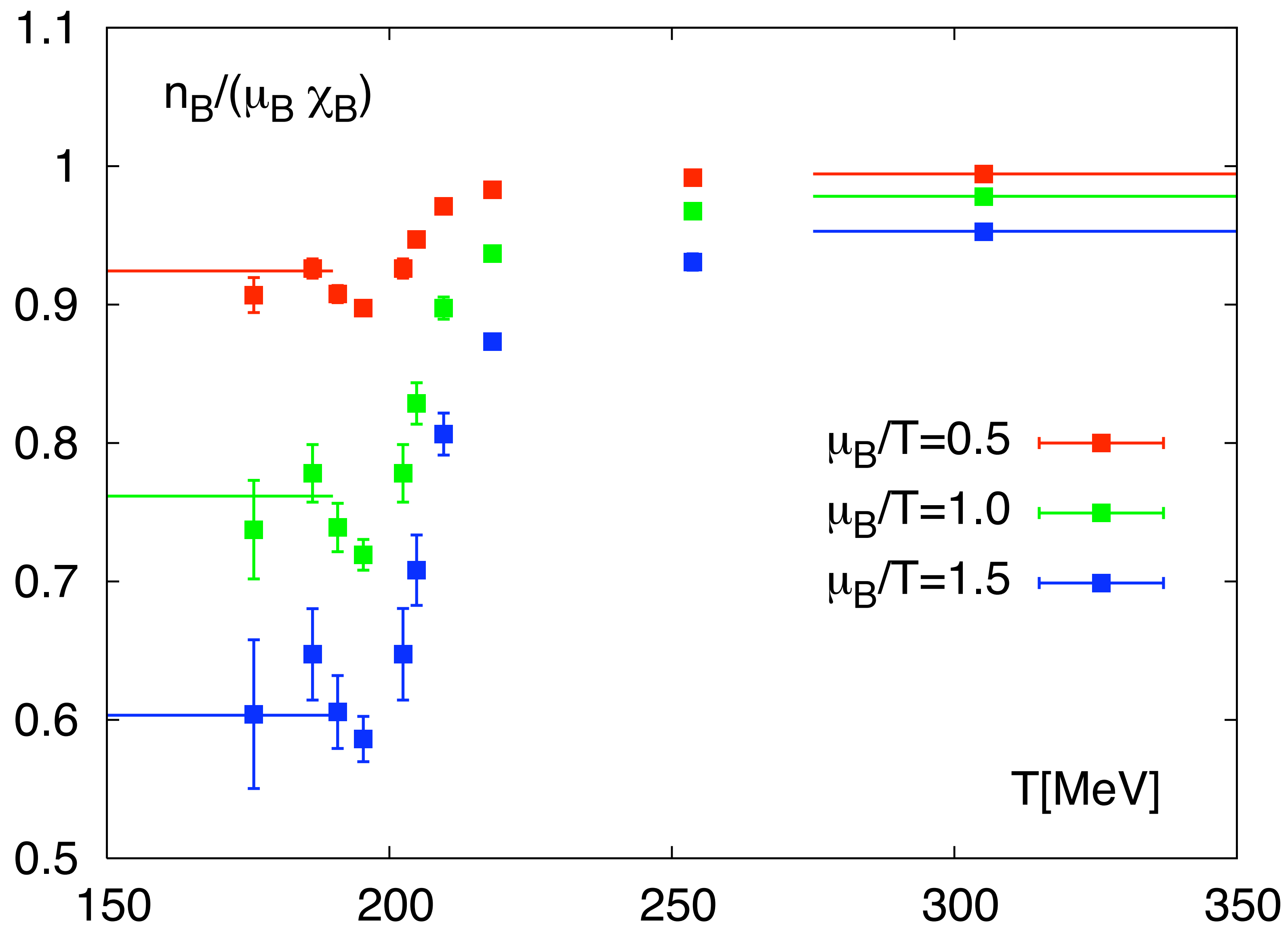




Compressibility



Isentropic Trajectories